Tax Evasion on a Social Network

Duccio Gamannossi Degl'Innocenti[1,2] Matthew D. Rablen[2,3]

[1]Tax Administration Research Centre

[2]University of Exeter

[3]University of Sheffield

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Introduction

- Tax evasion causes **significant losses of public revenues** (£4.4 bn. in UK)
- Growing interest among tax authorities in **how social attitudes to tax evasion are formed**
- "Big data" information systems potentially allow tax authorities to **perceive social networks to an unprecedented degree**
- Predictive tools find **patterns in data arising due to the determinants of subjects' decisions**
- We investigate the **impact of social network on tax evasion decisions** and develop a framework to **asses the value of social network data**
  - Is it worthwhile for a tax authority to invest in this technology?

Literature

- **Standard model** of tax evasion treats it as a **private decision**
- **More recent** work allows for **social interactions** to affect compliance
  - Social norms (Myles and Naylor, 1996)
  - Information updating (Hashimzade ***et al***., 2014)
  - Concern for relative consumption (Goerke, 2013)

Limitations of Existing Literature

- Taxpayers typically assumed to know aggregate-level statistics, e.g.,
  - **Proportion** of taxpayers who report honestly
- Implicitly **presupposes** that the **network is the complete** network
  - but taxpayers may only be able to access **heterogeneous ”local” information**
- Complete network also rules out
  - heterogeneity in social connectedness
  - highly-observed ”celebrity” taxpayers

Limitations of Existing Literature

- Other papers relax the complete network, but maintain other rigidities, e.g.,
  - **Undirected network** (so that a link from $\imath$ to $\jmath$ is necessarily reciprocated)
  - **Regular** toroidal **networks** with fixed patterns of connectivity
- For all these reasons, the **social networks so far used** seem to **deviate importantly from real-world networks**

Contribution

- We study a model allowing for an **arbitrary network**
- **Local relative consumption externalities**, heterogeneous across taxpayers
- Theoretical underpinnings to **network equilibria**

Research Questions

Our analysis has focused on **two** questions:

1. Is it possible to characterize **optimal evasion** in presence of relative utility and how do **social interactions** affect it?
2. How much does the **availability of more information** (especially related to social network) improves the capacity of a tax authority to **infer audit revenue effects**?

Preliminaries

- A taxpayer $\imath$ has true income $W_\imath$ on which they **should pay tax** $\theta(W_\imath)$.
- Taxpayer **may choose to evade** an amount of tax $E_\imath \in (0, \theta(W_\imath))$
- Evasion is a **risky** activity:
  - The **tax agency** is actively seeking to detect and **shut-down** evasion
  - There is a compound probability $p_\imath$ that:
    - **The taxpayer is discovered** under declaring
    - **The tax agency is successful** in shutting down evasion
- The tax authoritiy levies a **fine** $f > 1$ proportional to the evaded tax debt upon successful action
- Taxpayers care about **relative utility**
  - they benchmark consumption against a reference level R

The taxpayer's problem

$$\max_{E_\imath} \mathbb{E}\left(U_\imath\right) \equiv \left[1 - p_\imath\right] U\left(C_\imath^n - R_\imath\right) + p_\imath \left[U\left(C_\imath^a - R_\imath\right)\right]$$

*After-tax income **if not audited***

$$C_\imath^n \equiv X_\imath + E_\imath$$

*After-tax income **if audited***

$$C_\imath^a \equiv C_\imath^n - fE_\imath$$

Utility is linear-quadratic

$$U(z) = z[b_\imath - \tfrac{a_\imath z}{2}]$$

The **Privately Optimal Evasion** at an interior solution is:

$$E_\imath = \tfrac{1-p_\imath f}{a_\imath \zeta_\imath}\{b_\imath - a_\imath[X_\imath - R_\imath]\}$$

$$\zeta_\imath = \left[1 - p_\imath f\right]^2 + p_\imath \left[1 - p_\imath\right] f^2 > 0$$

Endogenising Reference Consumption

- **Observability of consumption** summarised by a **directed network** (graph), where a link (edge) from taxpayer (node) $\imath$ to taxpayer $\jmath$ indicates that $\imath$ observes $\jmath$'s consumption

- Links are **subjectively weighted**
  - some members of the reference group may be more focal comparators

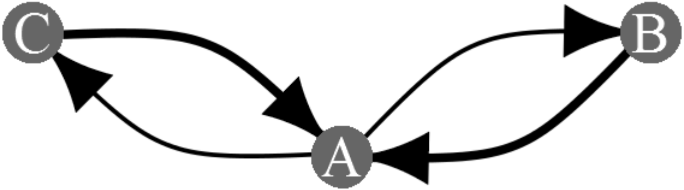
- **Network** of links is represented as an $N \times N$ (adjacency) matrix, $\boldsymbol{G}$, **of subjective comparison intensity weights** $g_{\imath\jmath} \in [0, 1]$,

- The weights satisfy

$$g_{\imath\imath} = 0; \qquad \sum_{\jmath \in \mathcal{R}_\imath} g_{\imath\jmath} = 1$$

- The **set of taxpayers** whose consumption is **observed** by taxpayer $\imath$ is termed $\imath$'s **reference group**, $\mathcal{R}_\imath$

An Example



$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & .5 & .5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \equiv \boldsymbol{G}$$

Endogenising Reference Consumption

- Reference consumption taken as the **weighted average of expected consumption** over the members **of the taxpayer reference group** $\mathcal{R}$

$$R_\imath = \sum_{j \in \mathcal{R}_\imath} g_{\imath\jmath} \mathbb{E}\left(\tilde{C}_\jmath\right)$$

Where:

$$\begin{aligned} \mathbb{E}\left(\tilde{C}_\jmath\right) &= [1 - p_\jmath]\, C_\jmath^n + p_\jmath C_\jmath^a \\ &= X_\jmath + [1 - p_\jmath f]\, E_\jmath \end{aligned}$$

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Nash Equilibrium – Bonacich Centrality

- **Network centrality** is a concept developed in sociology to quantify the **influence or power** of actors in a network
- **Multiple definitions**: Bonacich centrality (Bonacich, 1987) relevant in our setting

Definition

Consider a network with (weighted) adjacency matrix $\mathbf{A}$. For a scalar β and weight vector α, the weighted Bonacich centrality vector is given by $\mathbf{b}(\mathbf{A}, \beta, \alpha) = [\mathbf{I} - \beta\mathbf{A}]^{-1}\alpha$ provided that $[\mathbf{I} - \beta\mathbf{A}]^{-1}$ is well-defined and non-negative.

- The weighted Bonacich centrality computes the (α-weighted) sum of paths originating from a taxpayer in the network
- Longer paths are discounted by the (geometric) factor β

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Nash Equilibrium

Proposition

If

(i) utility is linear-quadratic, $U_\imath(z) = \left[b_\imath - \frac{a_\imath z}{2}\right] z$, with $a_\imath \in \left(0, \frac{b_\imath}{W_\imath}\right)$ and $b_\imath > 0$ for all $\imath \in \mathcal{N}$;

(ii) $1 > \rho(\boldsymbol{M})$; $\quad [\mathbf{I} - \boldsymbol{M}]\,\theta(\mathbf{W}) - \alpha > \mathbf{0}$;

then there is a unique interior Nash equilibrium, at which the optimal amount of tax evaded is given by

$$\mathbf{E} = \mathbf{b}(\boldsymbol{M}, 1, \alpha),$$

where

$$\begin{aligned} m_{\imath\jmath} &= \frac{[1 - p_\imath f][1 - p_\jmath f]}{\zeta_\imath} g_{\imath\jmath}; \\ \alpha_{\imath 1} &= \{[1 - p_\imath f] / [a_\imath \zeta_\imath]\}\{b_\imath - a_\imath [X_\imath - R_\imath(\mathbf{X})]\}. \end{aligned}$$

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Generalization of optimal evasion result

What **if utility is not linear-quadratic**?

For an **arbitrary** twice differentiable **utility function**
considering the FO linear approximation around a Nash equilibrium to the set of FOC, it is:

$$\mathbf{E} = \mathbf{J}\mathbf{E} + \widehat{\boldsymbol{\alpha}} = [\mathbf{I} - \mathbf{J}]^{-1}\,\widehat{\boldsymbol{\alpha}}$$

Where $\mathbf{J}$ is a matrix of coefficients measuring actions' interactions

A solution is a again in the form of a
weighted Bonacich centrality measure

Comparative Statics: Local Strategic Complementarity

- The model exhibits **strategic complementaries in evasion choices**
  - an increase in evasion by one taxpayer induces others to do likewise.
- Formally, **expected utility is supermodular in cross evasion choices**:

$$\frac{\partial^2 \mathbb{E}\left(U_\imath\right)}{\partial E_\imath \partial E_\jmath} = a_\imath g_{\imath\jmath}[1 - p_\imath f][1 - p_\jmath f] > 0 \qquad \jmath \in \mathcal{R}_\imath$$

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Comparative Statics: Optimal Evasion

- How is optimal evasion impacted by information carried through the social network?

$$\frac{\partial E_\imath}{\partial W_\jmath} = b_{1\imath}\left(\boldsymbol{M}, 1, \frac{\partial \alpha}{\partial X_\jmath}\right) \geq 0;$$
$$\frac{\partial E_\imath}{\partial p_\jmath} = b_{1\imath}\left(\boldsymbol{M}, 1, \frac{\partial \boldsymbol{M}}{\partial p_\jmath}\mathbf{E} + \frac{\partial \alpha}{\partial p_\jmath}\right) \leq 0.$$

- Results can be strengthened to strict inequalities if $\boldsymbol{G}$ is *connected*

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The Value of Network Information

- **Observing links in social networks** ought to help tax authorities to **target better** their limited **audit** resources

- Can tax authorities observe links in social networks?
  - Some individuals – celebrities – for whom it is common knowledge that many people observe them
  - "big data"

- The UK tax authority (HMRC) uses a system known as "Connect"
  - cross-checks public sector and third-party information
  - system produces "spider diagrams" linking individuals to other individuals and to legal entities such as "property addresses, companies, partnerships

- IRS also known to have also invested in big data heavily
  - but much more reticent in revealing its capabilities

Audit targeting

- Tax authority chooses **audit targets conditional** on observing each taxpayers' self-reported **income declaration** ($d_\imath$)

- By definition

$$E_\imath = \theta\left(W_\imath\right) - \theta\left(d_\imath\right)$$

- So

$$d_\imath \equiv \hat{d}_\imath\left(\boldsymbol{G}\right) = \theta^{-1}\left(\theta\left(W_\imath\right) - E_\imath\left(W_\imath; \boldsymbol{G}\right)\right).$$

- We invert this function to obtain

$$W_\imath \equiv \hat{W}\left(d_\imath; \boldsymbol{G}\right) = \hat{d}_\imath^{-1}\left(d_\imath\right)$$

- This gives the true income $W_\imath$ of a taxpayer who optimally declares an income $d_\imath$.

Limited network information

- If tax authority observes $\boldsymbol{G}$ (and the remaining model parameters) it can use $\hat{d}_\imath^{-1}(d_\imath)$ to recover the true incomes
$$\hat{W}(d_\imath; \boldsymbol{G}) = W_\imath$$

- If the tax authority **does not perfectly observe $\boldsymbol{G}$,** but instead some (related) network $\boldsymbol{G}'$, **estimates** of the $W_\imath$ **will be incorrect**
$$\hat{W}(d_\imath; \boldsymbol{G}') \neq W_\imath$$

- Noise in the $\hat{W}$ feeds through into noise in the $\hat{E} = \theta(\hat{W}_\imath) - \theta(d_\imath)$

- Suppose the tax **authority observes only a subset of the links** in the network
  - $\kappa \in [0, 1]$ is the **probability** that the tax authority **observes a given link** in the social network
  - **Network observed** by the tax authority denoted $\boldsymbol{G}(\kappa)$ generated by randomly deleting links (with probability $1 - \kappa$)

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Audit targeting

- Audits targeted to the $100\bar{p}\%$ of taxpayers with the **highest** $\hat{E}$
  - Reminiscent of US "DIF score", and similar to UK audit selection rules
- Full-information auditing gives revenue (in tax and fines) $\mathfrak{R}_{\max} = \mathfrak{R}(G(1))$
- No-information (random) auditing gives $\mathfrak{R}_{RA} = f\boldsymbol{p}\boldsymbol{E}$
- Metric used to assess value of **social network information**:

$$\Psi(\kappa) \equiv \frac{\mathfrak{R}(\boldsymbol{G}(\kappa)) - \mathfrak{R}_{RA}}{\mathfrak{R}_{\max} - \mathfrak{R}_{RA}} \times 100.$$

The Social Network

- Utilise a class of **generative network models** developed in the natural sciences
- Networks generated by **incremental addition of nodes and edges to a seed** network

Two key processes:

1. ***node-degree*** (or ***preferential attachment***) process – makes the probability that a new taxpayer added to the network observes an existing taxpayer, $\imath$, a **positive function of** $\imath$'s **in-degree** (the number of taxpayers who already observe $\imath$)
2. ***node-fitness*** process – makes the probability that a new taxpayer added to the network observes an existing taxpayer, $\imath$, a **positive function of** $\imath$'s ***fitness*** (an exogenous and time-invariant characteristic of node $\imath$)

The Social Network

- At step s of the generative process consider a taxpayer $\imath$ with degree $\mathfrak{d}_{\imath s}$, and fitness $\eta_\imath > 0$. Entwine the node-degree and node-fitness processes by allowing the probability that taxpayer $\imath$ is observed by the taxpayer added at step s to be proportional to the product

$$\eta_\imath A\left(\mathfrak{d}_{\imath s}\right) \qquad A'\left(.\right) > 0$$

- Special cases of this approach include
  - Barabási-Albert: $\eta_\imath$ equal across taxpayers
  - Bianconi-Barabási: $A\left(\mathfrak{d}\right) = \mathfrak{d}$
- We generate a static network using the Bianconi-Barabási **fitness** model using $\eta_\imath = W_\imath$ and $A\left(\mathfrak{d}\right) = \mathfrak{d}^{\phi}$ $\qquad \phi < 1$

$$\Pi_\imath = \frac{\mathfrak{d}_{\imath s}^{\phi} W_\imath}{\sum_{\jmath \in \mathcal{N}} \mathfrak{d}_{\jmath s}^{\phi} W_\jmath}$$

<u>The resulting **static** social networks used in our simulations resembles the ones observed empirically</u>

Model functions and parameters

- Tax system is linear: $\theta(W) = \theta W$
- Power law distribution of income
- Baseline parameter values
  - $\phi = 0.43$ (***Sublinear Preferential attachment***; Pham ***et al.***, 2016)
  - $N = 200$
  - $a = 2$
  - $b = 80$
  - pf calibrated to achieve evasion of 10%

Predicted wealth

Lemma

Under a linear income tax, the income of a taxpayer who declares income optimally is given by

$$\hat{\mathbf{W}}\left(\mathbf{d};\boldsymbol{G}\right)=\mathbf{b}(\mathbf{V},\theta,\gamma),$$

where

$$\begin{aligned} v_{\imath\jmath} &= \frac{\zeta_\imath}{\xi_\imath}m_{\imath\jmath}; \qquad \xi_\imath=\left[1-\theta\right]\left[1-p_\imath f\right]+\theta\left\{1+\left[f-2\right]p_\imath f\right\}>0; \\ \gamma_{\imath\jmath} &= \frac{\left\{1+\left[f-2\right]p_\imath f\right\}\theta a_\imath d_\imath+b_\imath\left[1-p_\imath f\right]}{a_\imath\xi_\imath} \\ &\quad +\frac{\left[1-p_\imath f\right]R(\mathbf{X}-\theta\left[1-p_\imath f\right]\mathbf{d})}{\xi_\imath}. \end{aligned}$$

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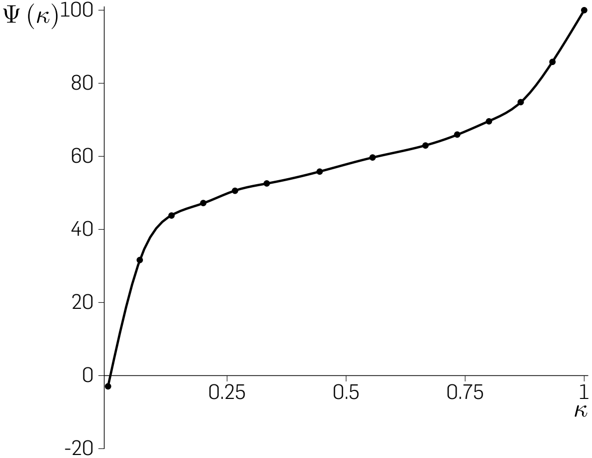
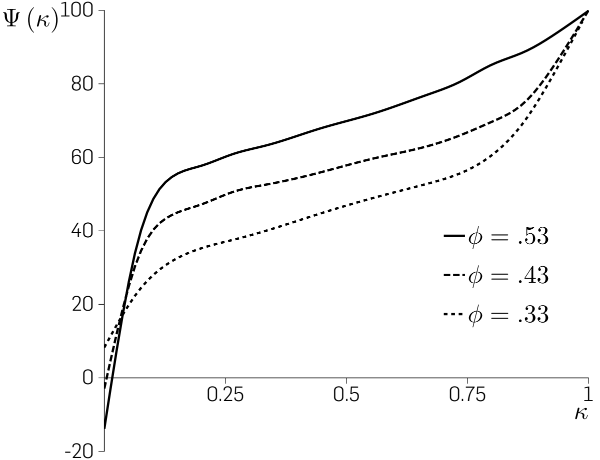
Findings – Baseline effects

- **Initial efforts** in collecting network information are characterized by **high returns**



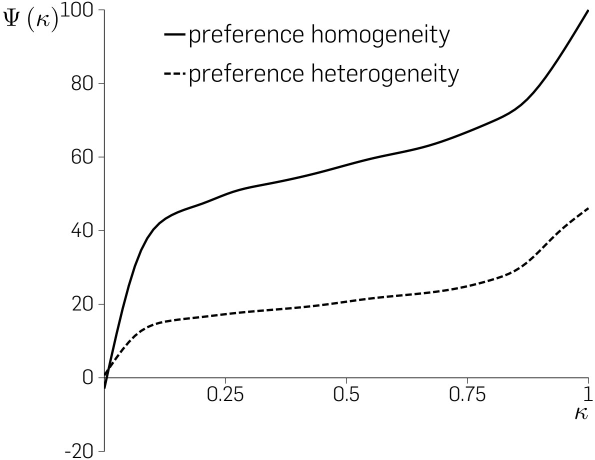
Findings – Effects of network structure

- The value of network information is **higher if preferential attachment ϕ is stronger**
- Using **predictive tools when little is know may be counterproductive** in highly concentrated networks



Findings – Effects of unobserved preference heterogeneity

- **Limited interaction** between uncertainty over **preference** and uncertainty over the **network**



Conclusions

- Our model provides a rich framework for understanding how information conveyed through a (arbitrary) social network influences optimal evasion behavior
- We show that **network information can be of value** to a tax authority
  - **strong gains to knowing a little** about the social network
  - **may actually be counterproductive** to utilize highly incomplete network information
- Some network information is **especially important in highly concentrated networks**

Further Research

- Introduce **habit** (memory) dependence in reference income
  - Investigate **dynamic response** to audit interventions
  - Study **direct and indirect effects** of audit interventions
- Extend the analysis to **avoidance** and **crime** as a whole
- Analyse how adding or **removing taxpayers** from the network (detention) may affect compliance

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Thank You!

Questions?