# Tax Avoidance and Evasion in a Dynamic Setting*

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**Abstract**

We study tax avoidance and tax evasion in an intertemporal utility maximization problem where evasion is fined if discovered, while avoidance is costly but entails a reduced payment upon audit (avoidance premium).

We find that traditional deterrence instruments (fine and frequency of audit) reduce optimal evasion but, in contrast with results in a static framework, they have no impact on optimal avoidance. Instead, tax avoidance depends negatively on its marginal cost and positively on both the tax rate and the avoidance premium. Our model shows that non-compliance behavior may result in a Laffer curve for fiscal revenues and that the revenue maximizing tax rate is lower the higher the avoidance premium. We characterize the optimal level of the avoidance premium by taking into account different government objectives: minimizing evasion, minimizing non-compliance (evasion plus avoidance), and maximizing revenues. Our results suggest that specific policies need to be implemented in order to deter avoidance (e.g., tax simplification) and we illustrate their impact on evasion.

**Keywords**: Tax Avoidance; Tax Evasion; Dynamic Programming; Tax Simplification

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**Abstract**

# 1 Introduction

To reduce tax burden, individuals may use a mix of tax evasion and tax avoidance. In Europe, tax evasion is about 20% of GDP, accounting for a potential loss of about 750-900 billion Euros each year (Buehn and Schneider, 2012; Murphy, 2019), i.e. about 13.2% of total revenue (Albarea et al., 2020). Intentional under-reporting of income is about 18-19% of the total reported income in the US, leading to a tax gap of about 500 billion dollars (Feige and Cebula, 2011), but the latter may increase to about 1 trillion dollars when tax avoidance is taken into account (Davison, 2021). Since the revenue loss is only the tip of the iceberg for what concerns the effect of tax evasion (Slemrod, 2007; Alm, 2012; Dzhumashev and Gahramanov, 2011; Markellos et al., 2016), reducing non-compliance is a priority for many governments, both in developed and developing countries.

Most western tax systems (e.g. Internal Revenue Service, 2014; European Parliament, 2017; HMRC, 2019) differentiate between tax evasion (illegal) and tax avoidance (legal),[1] and the academic literature has largely followed the same distinction (see, e.g. Slemrod and Yitzhaki, 2002; Sandmo, 2005; Wang et al., 2020). In this paper, we study evasion and avoidance jointly as they are similar in terms of depletion of revenue yields. However, the two activities differ in their level of sophistication, with the more refined avoidance representing a costly, but safer concealment option relative to evasion.

Our model is cast in a dynamic framework where we jointly compute the optimal evasion and avoidance, contrary to most of the literature which is set in a static (timeless) framework and disentangles the study of the two phenomena.

We study the dynamic programming problem of a representative consumer who maximizes the expected utility of his/her inter-temporal consumption, and decides the optimal percentage of evasion and avoidance. The consumer receives utility only from the consumption that exceeds a minimum (subsistence) amount at each period and his/her utility increases with the consumption of both a private and a public produced good. The agent is endowed with a linear $Ak$ technology and a constant tax rate is levied on the yield produced. The only source of uncertainty in the model is the audit which is performed randomly by the government. In particular, the agent knows the frequency of the audit but does not know the moment when it will be performed. When an audit happens, we assume that both evasion and avoidance are detected. Evasion is costless, but a fine must be paid upon detection, while avoidance is expensive but grants a reduction of the payment (fine plus tax liabilities) in case of detection. The share of the payment saved (the avoidance premium) depends on a parameter that, in our model, represents, in some way, the specific tax system and tax administration of the economic framework.

Our analysis shows that optimal avoidance is constant across time, it does

[1] While in the US avoidance has a neutral meaning, being understood as the use of tax regimes to one's own advantage to reduce one's tax burden, in several European countries (e.g., UK, Italy) the term has a negative connotation that implicitly assumes that the activity exploits an interpretation of the law that the legislator never intended.

not depend on preference parameters and is not directly affected by traditional deterrence instruments, like the fine and the frequency of audits. The extent of avoidance simply depends on (i) the tax rate (positively), (ii) the avoidance premium (positively), and (iii) the cost of avoidance (negatively). However, avoidance has a negative impact on tax evasion and, thus, total concealing crucially depends on both traditional fiscal parameters and tax avoidance determinants.

We find that the impact of the tax rate on total non-compliance share is not monotonic, and depends on the assumptions about the agent's preferences. In particular, when the agent receives utility only from consumption that exceeds a subsistence level, the relationship between evasion and the tax rate is ambiguous, while the two variables are inversely related for a zero subsistence level. Under this last condition, for low levels of either avoidance or taxation, an increase in the tax rate leads to a net improvement in compliance (the reduction in evasion is greater than the increase in avoidance), while the opposite holds true when the level of either avoidance or tax rate is high. We observe that a reduction in the avoidance premium reduces tax evasion when the avoidance premium is already low, while it may increase evasion otherwise. Moreover, an increase in evasion is more likely when minimum consumption is relatively high.

We show how non-compliance behavior may result in a Laffer curve for fiscal revenues, thus providing a theoretical explanation to a phenomenon documented by policymakers (Papp and Takáts, 2008; Vogel, 2012). We also find the revenue maximizing tax rate to be lower the higher the avoidance premium, a worrisome result for tax authorities given the expansion of the mass-marketed avoidance schemes targeted at middle income individuals that occurred in recent years (HMRC, 2020, 2021).

The article is structured as follows: Section 2 discusses our definitions of avoidance and evasion and reviews the related literature. Section 3 outlines the model. The optimization problem for the consumer and some possible government goals are analyzed in Sections 4 and 5. Section 6 discusses the results and their policy implications, while Section 7 draws some conclusions. The proofs are provided in the appendixes.

## 2 Tax avoidance and tax evasion in a dynamic framework

The first studies on tax compliance (e.g. Allingham and Sandmo, 1972, Yitzhaki, 1974) adapted Becker (1968)'s model about crime to tax evasion. Since then, an extensive microeconomic literature has been exploring: (i) taxpayers behavior (along with their psychological and sociological motivations), (ii) the impact of legislation and enforcement, and (iii) the availability of different non-compliance activities (Sandmo, 2005; Kirchler, 2007; Slemrod, 2007; Alm, 2019; Slemrod, 2019). Probably due to the formal legality of avoidance (Cross and Shaw, 1982), most of these initial studies totally neglected it, and the subsequent literature has largely retained this bias. Nevertheless, in spite of the scarcity of the literature about tax avoidance, the corresponding loss of tax revenue seems significant in many countries.[2]

Cross and Shaw (1981, 1982) are the first to jointly study evasion and avoidance[3], pointing out that: (i) for taxpayers they may be either substitutes or complements, and (ii) tax authorities must take into account both channels of response to their deterrence activities.

In Alm (1988a), the taxpayer optimally allocates an exogenous income between declaration, evasion, and avoidance. While evasion is modelled as a costless and risky asset following Yitzhaki (1974), avoidance is assumed to be costly but risk-less, granting a legal reduction of tax liabilities. Alm (1988a) shows that some degree of complexity in the cost of avoidance might be optimal for a government seeking to maximize either the net revenues or the social welfare function.

Alm et al. (1990) empirically study the taxpayers choice on how much to report, evade, and avoid. They find avoidance to be a substitute for both evasion and reported income, which are shown to be complements. The authors conclude that, in the light of this complex behavioral response, any evaluation of a tax system reform should be performed considering all the non-compliance opportunities.

Alm and McCallin (1990) assume both evasion and avoidance to be risky activities. Based on the portfolio theory, their analysis shows similar results to Alm (1988a), except for the impact of the fine which becomes unambiguously improving for government revenue. Assuming risk-less and visible avoidance, Cowell (1990) shows that a relatively high avoidance cost leads to polarization between poor and rich taxpayers, with the latter being the only ones who can afford avoidance.

In Gamannossi degl'Innocenti and Rablen (2016) the taxpayer is assumed to be rationally bounded, and chooses sequentially how much to invest in (risky) avoidance and evasion. The authors show that evasion might become an increasing function of the audit probability when the latter is low enough, yet tax avoidance is always decreasing in the probability of audit. Moreover, they show that a negative relationship between tax rate and evasion (so called Yitzhaki, 1974 puzzle) holds for avoidance.

Gamannossi degl'Innocenti and Rablen (2017b) study optimal tax enforcement when (risky) avoidance and evasion can simultaneously be performed. The authors find that a taxpayer's preferred mix of avoidance and evasion moves in favor of avoidance as reported income decreases and as the competitiveness of the market for avoidance schemes increases.

All the above mentioned studies assume a static modeling setting, although the tax evasion is, by its own nature, taken in a dynamic context, in which the consequences of a present action may affect future income (Wen-Zhung and

[2]Estimates provided by the UK tax authority put the value of tax avoidance at £1.7 bn, compared to £4.6 bn for tax evasion (HMRC, 2020). Lang et al. (1997) estimate that tax avoidance costs the German exchequer about 34% of income taxes paid.

[3]The model proposed by Cross and Shaw (1982, 1981) has been formalized in Slemrod (1989), where it is used to investigate the impact of fiscal complexity on compliance.

Yang 2001; Dzhumashev and Gahramanov 2011; Levaggi and Menoncin 2012).

Studies using the dynamic framework have recently shed light on the impact of uncertainty over fiscal parameters on evasion and growth Bernasconi et al. (2015), on the relationship between evasion and investment choices Levaggi and Menoncin (2016b), and on the role of habit in consumption when tax evasion can be performed Bernasconi et al. (2019).

In this work, we develop a model that studies the joint avoidance and evasion decision (e.g., Alm, 1988b; Alm and McCallin, 1990) in a dynamic setting, which is, in our opinion, the natural framework where these decision should be studied. Moreover, our model provides a more general specification for the penalty on tax avoidance that encompasses, as special cases, the ones considered in the literature so far.

# 3 The model

We model the choices of a representative consumer who maximizes his/her intertemporal utility that depends on the consumption of a private good ($c_t$) and a merit good ($g_t$) which is financed by a linear income tax. Our assumption is justified because, starting from the inception of the welfare state, the supply of goods like health care, education, and other personal services, have been increasing over time to become one of the biggest share of public expenditure in western countries (OECD, 2019). We assume that the agent is affected by fiscal illusion, which means that s/he does not perceive the link between public good provision and income tax so that s/he may engage in tax evasion and tax avoidance without internalizing the consequences of such behavior on the future supply of $g_t$.

## 3.1 Capital accumulation

The consumer is endowed with an initial capital $k_{t_0}$ that is used over the period $[t_0, \infty[$ to produce an income $y_t$ through the deterministic production function

$$y_t = Ak_t, \tag{1}$$

where $A$ measures total factor productivity. Since the stock of capital cannot produce an aggregate income greater than the capital itself, it is reasonable to assume that $0 < A < 1$. Although $A$ is exogenous and deterministic, the process of capital accumulation is endogenous because of the individual's consumption choice ($c_t$), and it is also stochastic due to the choices related to tax evasion ($e_t$) and tax avoidance ($a_t$).

Government levies a linear tax $0 \leq \tau \leq 1$ on income, which is used to finance the provision of the merit good ($g_t$). Without evasion, the net change in capital is

$$dk_t = ((1 - \tau) y_t - c_t) dt. \tag{2}$$

The agent assumes that $g_t$ does not depend on the income tax s/he pays. For this reason, s/he may try to reduce his/her tax burden by either evading a percentage ($e_t$) of the yield or by eroding his/her tax base through avoidance ($a_t$) whose effectiveness depends on the vulnerability of the tax system. When the agent is caught reducing the tax base, s/he has to pay a fine $\eta$ that is proportional to the hidden part of the total tax; however, in case of avoidance, a percentage $\beta$ of the fine is not paid. Accordingly, the total fine that must be paid in case of an audit is

$$\eta \left(e_t + \left(1 - \beta\right) a_t\right) \tau A k_t.$$

The parameter $\beta$ is a kind of premium that avoidance grants with respect to evasion when the taxpayer is audited. This parameter provides a measure of the vulnerability of the tax system to tax avoidance, and is lower in economies in which: (i) tax codes are simpler and less ambiguous, (ii) tax authorities are endowed with relatively sizable operational[4] and litigation resources, and (iii) courts have higher effectiveness. If this premium is maximum (i.e. $\beta = 1$), then avoidance is risk-less (like in Alm, 1988b). Instead, for $\beta = \frac{\eta - 1}{\eta}$ the taxpayer's payment conditional on audit is the same as for true income reporting (like in Alm and McCallin, 1990). This implies that, for any level above this threshold, the consumer gets an actual discount on his/her tax bill even in case of audit, while if $\beta$ is below the threshold the consumer reduces the fines s/he has to pay if caught.

Contrary to other works (Chen, 2003; Dzhumashev and Gahramanov, 2010), we assume tax evasion to be a cost-less activity. Conversely, avoidance is assumed to be expensive since a considerable effort[5] (or expertise[6]) is needed to reduce the tax burden while not violating the law.

To keep our results as general as possible, the costs of avoidance are represented through any increasing and convex function $f\left(a_t\right)$.[7] Notably, this formulation allows accounting for the (likely) occurrence of fixed costs $f(0) \geq 0$ (setup costs, e.g. creation of legal entities) and has the flexibility to represent any mix of avoidance instruments.

In line with Levaggi and Menoncin (2012, 2013); Bernasconi et al. (2015); Levaggi and Menoncin (2016a,b), we model the audit process through a Poisson jump process $d\Pi_t$ whose frequency is $\lambda dt$ which coincides with the two first moments of the jump

$$\mathbb{E}_t \left[d\Pi_t\right] = \mathbb{V} \left[d\Pi_t\right] = \lambda dt.$$

[4]In a recent paper, Guyton et al. (2021) show that more detailed/thorough audits are able to uncover avoidance activities that are mostly missed by standard random audits.

[5]People display a limited understanding of tax law and tax rates, see Blaufus et al. (2015) and Stantcheva (2021).

[6]The avoidance schemes are often complex, see the discussion in Li et al. (2021).

[7]Evidence on the contractual terms upon which avoidance schemes are typically sold is scarce. However, Committee of Public Accounts (2013) reports that the majority of mass-marketed schemes entail a fee related to the reduction in the annual theoretical tax liability of the user and HMRC (2015) reports that fees vary with the value of the amount of the investment realized by the scheme.

Thus, the final dynamics of capital $k_t$ is

$$\begin{aligned} dk_t = & \left(Ak_t - \tau\left(1 - e_t - a_t\right) Ak_t - c_t - f\left(a_t\right) Ak_t\right) dt \\ & - \eta\left(e_t + \left(1 - \beta\right) a_t\right) \tau Ak_t d\Pi_t, \end{aligned} \tag{3}$$

since the tax $\tau$ is paid only on the income that is not hidden $(1 - e_t - a_t)$, and the avoidance cost $f(a_t)$ is proportional to income.

The expected value of $dk_t$ is

$$\mathbb{E}_t\left[dk_t\right] = \left(\left(1-\tau\right) Ak_t + \left(1-\eta\lambda\right) e_t \tau Ak_t + \left(1 - \eta\lambda\left(1-\beta\right) - \frac{f\left(a_t\right)}{a_t \tau}\right) a_t \tau Ak_t - c_t\right) dt,$$

from which we see that evasion is expedient on average if

$$\mathbb{E}_t\left[dk_t\right] > \mathbb{E}_t\left[dk_t\right]_{e_t=0},$$

which becomes

$$1 - \eta\lambda > 0, \tag{4}$$

and, accordingly, we will assume that the product between the frequency of audit $(\lambda)$ and the fine $(\eta)$ is lower than 1. Instead, avoidance is expedient on average if

$$\mathbb{E}_t\left[dk_t\right] > \mathbb{E}_t\left[dk_t\right]_{a_t=0},$$

which becomes

$$\frac{f\left(a_t\right)}{a_t} < \frac{f\left(0\right)}{a_t} + \left(1 - \eta\lambda\left(1-\beta\right)\right)\tau.$$

Hence, the individual will engage in avoidance if its costs are lower than a threshold dependent on both fixed avoidance cost, and fiscal and enforcement parameters.

Since the product $\eta\lambda$ is higher than 1 for an expedient evasion, then the minorant of the right hand side is $\beta\tau$, and so we can impose that

$$f\left(a_t\right) < f\left(0\right) + a_t \beta\tau, \tag{5}$$

in which we further assume that $f(0) < 1 - \tau$.

## 3.2 Consumer's preferences

The representative agent receives utility from consuming both a private produced good $(c_t)$ and a public produced good $(g_t)$, and we assume that such a utility is additive in these two goods.

The agent's behavior is described by a Hyperbolic Absolute Risk Aversion utility (see, for instance, Gollier, 2001) written on the instantaneous consumption as

$$U\left(c_t\right) = \frac{\left(c_t - c_m\right)^{1-\delta}}{1-\delta} + v\left(g_t\right), \tag{6}$$

where $c_m$ is a minimum (subsistence) amount of consumption that the agent needs to consume, the parameter $\delta > 0$ measures the risk aversion, and $v(\bullet)$ is an increasing and concave function. The existence of a strictly positive subsistence consumption level allows us to solve some puzzles and to reconcile theoretical findings with empirical evidence (see, for instance, Sethi et al. 1992; Weinbaum 2005; Achury et al. 2012 for the role of subsistence consumption in portfolio choice and Strulik, 2010 for its role in modeling economic growth). The Arrow Pratt absolute risk aversion index is

$$-\frac{\frac{\partial^2 U(c_t)}{\partial c_t^2}}{\frac{\partial U(c_t)}{\partial c_t}} = \frac{\delta}{c_t - c_m}, \tag{7}$$

which increases when either $\delta$ or $c_m$ increase. In other words, a consumer with low $\delta$ but whose consumption is closer to $c_m$ behaves exactly as a consumer with a higher $\delta$ but a consumption level farther from $c_m$.

# 4 The Problem

If the agent discounts future utility at a constant rate $\rho$, the optimization problem can be written as

$$\max_{\{c_t, e_t, a_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \frac{(c_t - c_m)^{1-\delta}}{1-\delta} e^{-\rho(t-t_0)} dt\right], \tag{8}$$

under the capital dynamics (3).

**Proposition 1.** *The optimal solution to Problem (8), given the capital dynamics (3), is*

$$a^* = (f')^{-1}(\tau\beta), \tag{9}$$

$$e_t^* = \frac{k_t - H}{\tau\eta A k_t}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) - (1-\beta)\, a^*, \tag{10}$$

$$c_t^* = c_m + (k_t - H)\left(\frac{\rho+\lambda}{\delta} + \frac{\delta-1}{\delta}\frac{1}{\eta} + \frac{\delta-1}{\delta}(1-\tau)\,A - \frac{1}{\eta}(\lambda\eta)^{\frac{1}{\delta}} + \frac{\delta-1}{\delta}(\tau\beta a^* - f(a^*))\,A\right), \tag{11}$$

*in which $(f')^{-1}$ is the inverse of the first derivative of the function $f$, and*

$$H := \frac{c_m}{A(\tau\beta a^* - f(a^*) + (1-\tau))}.$$

*Proof.* See Appendix A. □

In the proposition above $H$ is a constant whose value coincides with the present value of a perpetuity. In fact, we can write

$$H = \int_t^{\infty} c_m e^{-A(\tau\beta a^* - f(a^*) + (1-\tau))(s-t)} ds,$$

which is always positive because of condition (5). Thus, we can conclude that $H$ represents the total present value of the future subsistence consumption $c_m$, discounted at a rate given by the total factor productivity corrected by both the tax rate and a function of avoidance. Accordingly, $k_t - H$, can be considered as the disposable capital that remains after saving enough for financing the future streams of subsistence consumption.

We note that when avoidance is not expedient (i.e. $a^* = 0$), the discount rate is given by the total factor productivity net of tax and fixed avoidance costs: $A\left((1-\tau) - f(0)\right)$. We can immediately check that, under condition (5), the presence of avoidance (i.e $a^* > 0$) increases optimal consumption.

Optimal tax avoidance is constant across time and does not depend on the preference parameters (consumer's risk aversion) nor on the audit parameters $\eta$ (the fine) and $\lambda$ (the frequency of controls). It simply depends on its cost (the shape of the function $f(\bullet)$), the vulnerability of the tax system to avoidance $\beta$, and the tax rate $\tau$. The only fiscal parameter that affects tax avoidance is the tax rate, while, in sharp contrast with the static literature (Alm, 1988b; Alm et al., 1990; Gamannossi degl'Innocenti and Rablen, 2016, 2017a), classical evasion deterrence instruments are ineffective. The share of income that is avoided also does not depend on risk preferences: if the system allows for some loopholes, taxpayers will always use them to reduce their tax burden.

Tax evasion, on the contrary, is used as a "top up" to tax avoidance even if the "substitution rate" is not one. The share of evaded income depends on the fiscal parameters and is similar to the optimal tax evasion of other dynamic models (Levaggi and Menoncin 2013). In our setting, we show that tax avoidance, while reducing evasion, increases total hidden revenue, i.e., the sum of optimal evasion and avoidance:

$$E_t^* = e_t^* + a^* = \frac{k_t - H}{\tau \eta A k_t}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) + \beta \left(f'\right)^{-1}(\tau\beta). \tag{12}$$

The existence of a subsistence level of consumption implies that optimal evasion is time dependent as shown in Proposition 1. In the following corollary, we show that with $c_m = 0$ evasion is constant over time and so is consumption share.

**Corollary 1.** *The optimal solution to Problem (8) for a CRRA consumer (i.e. $c_m = 0$), given the capital dynamics (3), is*

$$a^* = \left(f'\right)^{-1}(\tau\beta),$$

$$e^* = \frac{1}{\tau\eta A}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) - (1-\beta)\,a^*,$$

$$\frac{c_t^*}{k_t} = \frac{\rho+\lambda}{\delta} + \frac{\delta-1}{\delta}\frac{1}{\eta} + \frac{\delta-1}{\delta}(1-\tau)A - \frac{1}{\eta}(\lambda\eta)^{\frac{1}{\delta}} + \frac{\delta-1}{\delta}\left(\tau\beta a^* - f(a^*)\right)A.$$

*Proof.* It is sufficient to set $c_m = 0$ in Proposition 1. □

The results in Proposition 1 and Corollary 1 allow drawing some interesting conclusions on the dynamic path of the variables of choice of the consumer.

While the optimal share of avoided income is fixed, the dynamics of consumption, capital, and tax evasion is more nuanced and we investigate it graphically in Figure 1.[8] When $c_m > 0$, these variables are affected by the (random) audits, so we perform $N = 1000$ replications and report the average (solid line) along with the zero and one quantiles (dashed lines). Panel a) shows that, for $c_m = 0$ (dot-dashed line), the evaded share of income is fixed in time due to the constant relative risk aversion. When $c_m > 0$, the evasion is increasing in time and in the long-run it tends to the its optimal level when $c_m = 0$. This dynamics of evasion is driven by the growth of consumer's consumption $c_t^*$ that reduces risk aversion $\frac{\delta}{c_t^* - c_m}$. Panel b) illustrates the evolution of consumption as a ratio of capital. The dynamics of $c_t^*/k_t$ when $c_m = 0$ is shown to be constant over time and lower than the case with $c_m > 0$. When $c_m > 0$, $c_t^*/k_t$ is decreasing over time due to a more sustained growth of the denominator and in the long-run it approaches its $c_m = 0$ level. Interestingly, the quantile lines show that, upon audit, the reduction in consumption is bigger than the one in capital.

— Insert Figure 1 about here —

Corollary 1 highlights that all the consumer's choice variables are time invariant when the agent does not ensure a positive minimum consumption. Indeed, the dynamic of $e_t^*$ and $c_t^*/k_t$ in Proposition 1 is due to a habit effect (induced by the minimum consumption) that leads to a gradual convergence towards the equilibrium levels of the case where $c_m = 0$.

## 4.1 Comparative statics

Here, we compute the behavior of the three variables $a^*$, $e_t^*$, and $E_t^*$ with respect to the model parameters.

Optimal avoidance $a^*$ increases with respect to both $\beta$ and $\tau$ as expected. This result matches the evidence in the empirical literature[9] and shows that the Yitzhaki's puzzle does not hold for tax avoidance in a dynamic setting, as proven in a static framework by Alm and McCallin (1990).[10]

From 10, optimal evasion decreases if $\tau$ increases, thus confirming the Yitzhaki's puzzle. In this case, the presence of avoidance reinforces the dampening effect

[8] In the figure we use the following specification: $k_0 = 100$, $A = 0.3$, $c_m = 10$, $\beta = 0.5$, $\delta = 2.5$, $\lambda = 0.3$,$\eta = 2.5$, $\rho = 0.05$, $\tau = 0.3$, $f(a_t) = a_t^\gamma$, $\gamma = 2$.

[9] Long and Gwartney (1987), Alm et al. (1990), and Lang et al. (1997) show that tax avoidance increases with the tax rate for US, Jamaican, and German households. As reviewed in Riedel (2018), the scientific literature unanimously reports evidence of substantial tax motivated profit shifting. Also Beer et al. (2020) perform a comprehensive meta-analysis of existing studies suggesting an elasticity of before tax income to corporate tax rate of minus one.

[10] Gamannossi degl'Innocenti and Rablen (2017b) show a similar result when the tax agency enforces the optimal truth telling probability, and so their result holds only for the individual with the highest level of avoidance in the system.

already observed when the fine is proportional to evaded taxes. The same effect can be observed for an increase in $\lambda$ and $\eta$. Instead, the reaction of $e_t^*$ to changes in $\beta$, measured by

$$\frac{\partial e_t^*}{\partial \beta} = \frac{\tau a^* A H^2}{c_m} \frac{e_t^* + (1-\beta) a^*}{k_t - H} + a^* - (1-\beta) \frac{\partial a^*}{\partial \beta},$$

is not trivial to compute. For $c_m = 0$ (i.e. $H = 0$), the optimal tax evasion may be either increasing or decreasing w.r.t. $\beta$:

$$\left.\frac{\partial e_t^*}{\partial \beta}\right|_{c_m=0} = a^* - (1-\beta) \frac{\partial a^*}{\partial \beta} \gtreqless 0 \iff \frac{\partial a^*}{\partial \beta} \frac{1}{a^*} \lesseqgtr \frac{1}{1-\beta},$$

i.e. evasion is increasing in $\beta$ only if the elasticity of $a^*$ w.r.t. $\beta$ is lower than a given threshold. This result has an interesting interpretation from a policy point of view: increasing the tax system robustness to avoidance (lowering $\beta$) reduces tax evasion if the avoidance premium is low, while it may increase evasion if the avoidance premium is high. This also means that for a system rather vulnerable to tax avoidance, a marginal decrease in the avoidance premium may worsen the evasion statistics. Other things being equal, the derivative for $\beta$ is higher when $c_m > 0$ relative to $c_m = 0$, meaning that the value of $\beta$ for which the derivative changes sign is lower in the former case. Finally, it is interesting to observe that if $c_m = 0$, the value of $\beta$ which minimizes the evasion must satisfy the condition

$$\frac{\partial a^*}{\partial \beta} \frac{\beta}{a^*} = \frac{\beta}{1-\beta}.$$

The total unreported revenue share (12) reacts to changes in $\tau$ in an ambiguous way because of the reduction in tax evasion and the increase in tax avoidance.[11] The derivative can be written as:

$$\frac{\partial E_t^*}{\partial \tau} = \underbrace{-\frac{1}{\tau \eta A k_t} \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) \left(H \frac{1 - \beta a^*}{\tau \beta a^* - f(a^*) + (1-\tau)} + \frac{k_t - H}{\tau}\right)}_{<0} + \underbrace{\beta \frac{\partial \left(f'\right)^{-1} (\tau\beta)}{\partial \tau}}_{>0},$$

and even for the simpler case with $c_m = 0$ the sign remains ambiguous:

$$\left.\frac{\partial E_t^*}{\partial \tau}\right|_{c_m=0} = \underbrace{-\frac{1}{\tau^2 \eta A} \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right)}_{<0} + \underbrace{\beta \frac{\partial \left(f'\right)^{-1} (\tau\beta)}{\partial \tau}}_{>0}.$$

Notably, when $\tau$ is relatively high, the negative term is smaller (in absolute value) and the positive one is bigger. This implies that an increase in tax rate reduces total reported income in economies with sufficiently high taxation.

[11] Notice that in the static framework, Alm and McCallin (1990) reports an increase in total non compliance, while Gamannossi degl'Innocenti and Rablen (2016) a reduction. The result in Alm (1988b) is analogous to ours, but it follows from the more general specification of the fine, tax and avoidance cost functions.

Finally, the derivative of $E_t^*$ with respect to $\beta$ can be written as:

$$\frac{\partial E_t}{\partial \beta} = \frac{1}{\tau \eta A k_t} \left(1 - (\lambda \eta)^{\frac{1}{\delta}}\right) \frac{H \tau a^*}{(\tau \beta a^* - f(a^*) + (1-\tau))} + a^* + \beta \frac{\partial a^*}{\partial \beta} > 0.$$

The comparative statics results derived in this section, along with the results on government revenue of the next section, are summarized in Table 1.

Table 1: Effect of enforcement/fiscal parameters on avoidance, evasion, and tax revenue. The table presents the sign of derivatives (null, positive, negative, or undetermined) of the function in the column with respect to the parameter in the row: sign $\left(\frac{\partial \mathrm{Col}}{\partial \mathrm{Row}}\right)$

| | $a^*$ | $e_t^*$ | $E_t^* = a^* + e_t^*$ | $\mathbb{E}_t[dT_t]$ |
|---|---|---|---|---|
| $\lambda$ | 0 | $-$ | $-$ | $+$ |
| $\eta$ | 0 | $-$ | $-$ | $+$ |
| $\beta$ | $+$ | und. | $+$ | $-$ |
| $\tau$ | $+$ | $-$ | und. | und. |

# 5 The optimal capital dynamics and government revenue

The dynamics of the optimal capital is[12]

$$\frac{d(k_t^* - H)}{(k_t^* - H)} = \frac{1}{\delta}\left((1-\tau)A - (\rho+\lambda) + \frac{1}{\eta} + (\tau\beta a_t^* - f(a_t^*))A\right) dt - \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) d\Pi_t.$$

We stress that the solution to the process $k_t^* - H$ is exponential, and is always positive if the initial value $k_{t_0}^* - H$ is positive. Thus, we can conclude that the optimal capital will never fall below the value $H$ if $k_{t_0}^* > H$. We recall that $H$ is the discounted present value of the future subsistence consumption levels $c_m$ and, accordingly, this result allows concluding that the agent behaves in such a way to guarantee that his/her capital is always able to finance the future flow of subsistence consumption.

The expected growth rate of the modified capital $k_t^* - H$ is

$$\gamma^* := \frac{1}{dt}\mathbb{E}_t\left[\frac{d(k_t^* - H)}{(k_t^* - H)}\right] = \frac{1}{\delta}\left((1-\tau)A - (\rho+\lambda) + \frac{1}{\eta} + (\tau\beta a_t^* - f(a_t^*))A\right) - \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right)\lambda.$$

If we compute the first derivative of $\gamma^*$ with respect to $\beta$ we get

$$\frac{\partial \gamma^*}{\partial \beta} = \frac{1}{\delta}\left(\frac{\tau}{\eta} a_t^* + \tau\beta \frac{\partial a_t^*}{\partial \beta} - f'(a_t^*)\frac{\partial a_t^*}{\partial \beta}\right) A,$$

[12] We recall that, since $H$ is constant, $d(k_t^* - H) = dk_t^*$.

and because of the first order condition $f'(a_t^*) = \tau\beta$, we can write

$$\frac{\partial \gamma^*}{\partial \beta} = \frac{1}{\delta}\frac{\tau}{\eta} a_t^* A > 0,$$

which is always positive. Therefore, it is optimal to choose the highest value for $\beta$ (i.e. $\beta^* = 1$). This result is not surprising: given the assumption of a merit good that does not increase private capital productivity, growth is maximized with a minimum tax revenue.

The dynamics of government inflows $dT_t$ is

$$dT_t = \tau(1 - e_t^* - a_t^*) y_t dt + \eta (e_t^* + (1-\beta) a_t^*) \tau y_t d\Pi_t,$$

which can be written as

$$dT_t = \left(\tau(1-\beta a_t^*) Ak_t - \frac{k_t - H}{\eta}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right)\right) dt + (k_t - H)\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) d\Pi_t,$$

whose expected value is

$$\mathbb{E}_t[dT_t] = \left[\tau(1-\beta a_t^*) Ak_t - \frac{k_t - H}{\eta}(1 - \lambda\eta)\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right)\right] dt.$$

When we compute the derivative of this expected value with respect to $\tau$ we get

$$\frac{1}{dt}\frac{\partial \mathbb{E}_t[dT_t]}{\partial \tau} = (1-\beta a_t^*) Ak_t - \tau\beta\frac{\partial a_t^*}{\partial \tau} Ak_t + \frac{1}{\eta}(1-\lambda\eta)\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) H \frac{1 - \beta a^*}{(\tau\beta a^* - f(a^*) + (1-\tau))},$$

whose sign may change as a function of $\tau$ itself. In particular, we see that when $c_m = 0$, this formula simplifies to

$$\left.\frac{1}{dt}\frac{\partial \mathbb{E}_t[dT_t]}{\partial \tau}\right|_{c_m=0} = (1-\beta a_t^*) Ak_t - \tau\beta\frac{\partial a_t^*}{\partial \tau} Ak_t,$$

from which we get:

$$\left.\frac{1}{dt}\frac{\partial \mathbb{E}_t[dT_t]}{\partial \tau}\right|_{c_m=0} \gtreqless 0 \iff \tau \lesseqgtr \frac{1 - \beta a_t^*}{\beta \frac{\partial a_t^*}{\partial \tau}}.$$

Our model entails a Laffer curve behaviour:[13] for $\tau$ sufficiently low, the revenue increases as $\tau$ increases because the rise in the tax rate, and the reduction of tax evasion, more than offset the increase in tax avoidance. However, as $\tau$ increases, the latter effect becomes prevalent and the revenue starts decreasing. Notably, the level of the revenue maximizing tax rate is inversely related to $\beta$: the higher the $\beta$, the lower the level of the tax rate for which an increase in the tax rate produces a decrease in the revenue.

[13] Refer to the Appendix B for additional details.

The reaction to government's expected inflows with respect to $\beta$ is

$$\frac{\partial}{\partial \beta}\left(\frac{1}{dt}\mathbb{E}_t\left[dT_t\right]\right) = -\tau a_t^* A k_t - \tau\beta\frac{\partial a_t^*}{\partial \beta}Ak_t - \frac{1}{\eta}\left(1-\lambda\eta\right)\left(1-\left(\lambda\eta\right)^{\frac{1}{\delta}}\right)\frac{1}{A}\frac{c_m \tau a^*}{\left(\tau\beta a^* - f\left(a^*\right) + \left(1-\tau\right)\right)^2}$$

whose sign is negative.

Finally, an increase in $\eta$ produces an increase in the expected revenue. Indeed, since $k_t - H > 0$,

$$\frac{\partial}{\partial \eta}\left(\frac{1}{dt}\mathbb{E}_t\left[dT_t\right]\right) > 0 \iff \left(1-\left(\lambda\eta\right)^{\frac{1}{\delta}}\right) + \left(1-\lambda\eta\right)\frac{1}{\delta}\left(\lambda\eta\right)^{\frac{1}{\delta}} > 0.$$

# 6 Discussion and policy implications

The results and the comparative statics presented in the previous sections highlight the importance of studying tax evasion and tax avoidance as a joint decision. The results of our model indeed show that several interesting policy implications can be derived from this analysis and that the institutional setting (especially $\beta$) may change the outcome of policies aimed at reducing non compliance. In what follows we summarize and discuss the most important results of our model.

1. Tax avoidance depends neither on investor's risk attitude nor on audit parameters (frequency and fines). This implies that government cannot alter the avoidance decision using ordinary tax enforcement tools (level of the fine and number of audits). Instead, this result can be obtained through an increase in the quality (litigation resources and thoroughness) of the audits or fiscal/legal reforms. However, avoidance deterrence might entail unintended consequences:

    (a) Even if evasion is decreasing in the tax rate, there are limits to the use of the tax rate as an instrument to improve compliance due to the presence of a Laffer curve on total Government revenue, which provides a theoretical explanation to a phenomenon documented by policymakers (Papp and Takáts, 2008; Vogel, 2012). This finding follows from the three effects induced by a rise in the tax rate: (i) a mechanical increase of revenues due to the higher marginal tax rate, (ii) a reduction in evasion, (iii) an increase in avoidance.

    (b) Policies aimed at increasing avoidance costs, while theoretically identifiable, seem to have a limited practical relevance. The costs to engage in avoidance are related to the effort (or expertise) required to have a deep understanding of the "loopholes" in the tax law. An increase in these costs entails a trade-off, as these costs also apply to "intended" economic activities. A more effective way to reduce tax avoidance is to reduce $\beta$, i.e. the avoidance premium, through a simplification of the tax system.[14] Investing in tax simplification, intended as the reduction of the extent of variation in possible tax treatments of economic activities (number of deductions, exemptions and instances of preferential treatment of income), has also been recommended in the literature (e.g., Skinner and Slemrod 1985; Mccaffery, 1990; Kopczuk, 2006) for its several desirable outcomes.

2. Our analysis shows that tax avoidance deterrence performed by changing the tax rate or the avoidance premium might entail an unwanted increase in tax evasion, which can however be sterilized by raising either the frequency of audit or the fine.

3. The opposite impacts of the tax rate on avoidance and evasion may provide an alternative interpretation for the so called Yitzhaki's puzzle. While, from a theoretical point of view, it is possible to disentangle evasion from avoidance, the distinction is much more complex in an empirical setting. An imperfect measure of tax evasion (which may include also a part of tax avoidance) would lead to a spurious estimation, as the recent IRS estimates on the tax gap show.

Over the last few decades, the most striking worldwide trend in tax policy has been the decline in corporate income tax rates. Some argue (e.g. Tørsløv et al. 2020) that this is an effect of the tax reduction performed in many countries to face the competition of tax heavens.

We show that a similar mechanism might also be at work for individual income tax: when avoidance is more profitable (higher $\beta$), the tax rate that maximizes government revenue and the revenues themselves are lower. This result has important policy implications: the expansion of mass-marketed avoidance schemes targeted at employees, professionals, and contractors (HMRC, 2020, 2021), has increased tax competition from tax heavens for personal income tax revenue. Our results suggest that anti-avoidance efforts of tax authorities/governments/international organizations should be extended to personal income to prevent future reduction in tax rates and revenues.

# 7 Conclusion

In this paper we developed what can be considered, to the best of our knowledge, the first dynamic model studying taxpayer's avoidance and evasion. Evasion is cost-less, but entails the payment of a fine if detected. Instead, avoidance is costly, but has a return premium relative to evasion upon audit.

Contrary to previous studies in a static framework, our results showed that optimal avoidance does not depend on audit parameters (frequency of the audits and fine to be paid when caught evading) in an intertemporal setting. Tax avoidance, unlike evasion, is also not affected by the risk preferences of the taxpayer. The share of avoided income results from a cost-benefit analysis:

[14] On specific anti avoidance reforms of the tax system, see Gravelle (2014)

the (certain) avoidance cost measures the (money equivalent) effort (or hired expertise cost) needed to engage in avoidance, while the (uncertain) benefit is the potential reduction of the fine to be paid when audited.

From a policy point of view, our model shows that reducing tax evasion may be a government objective that is rather different from maximizing revenue, especially in the presence of tax avoidance. Given the opposite impact of the tax rate on avoidance and evasion, we find that a Laffer curve exists between the tax rate and fiscal revenue. Our analysis also shows the importance of the avoidance premium and highlights its possible detrimental impact on evasion. In particular, a reduction in the avoidance premium leads to an increase of collected revenues but might entail a rise of tax evasion for economies more vulnerable to avoidance.

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# A Proof of Proposition 1

Given Problem (8), we can define the value function at any time $t \in [t_0, \infty)$ as

$$J(t, k_t) = \max_{\{c_s, e_s, a_s\}_{s\in[t,\infty[}} \mathbb{E}_{t_0}\left[\int_t^\infty \frac{(c_s - c_m)^{1-\delta}}{1-\delta} e^{-\rho(s-t)} ds\right],$$

and so the Hamilton-Jacobi-Bellman (HJB) equation is

$$0 = \frac{\partial J(t, k_t)}{\partial t} - (\rho + \lambda) J + \frac{\partial J(t, k_t)}{\partial k_t}(1-\tau) A k_t + \max_{c_t}\left[\frac{(c_t - c_m)^{1-\delta}}{1-\delta} - \frac{\partial J(t, k_t)}{\partial k_t} c_t\right]$$
$$+ \max_{e_t, a_t}\left[\frac{\partial J(t, k_t)}{\partial k_t}(\tau(e_t + a_t) A k_t - f(a_t) A k_t) + \lambda J(t, k_t - \eta(e_t + (1-\beta) a_t)\tau A k_t)\right].$$

The First Order Condition (FOC) with respect to $c_t$ is

$$c_t^* = c_m + \left(\frac{\partial J(t, k_t)}{\partial k_t}\right)^{-\frac{1}{\delta}}.$$

The FOC with respect to $a_t$ is

$$\frac{\partial J(t, k_t)}{\partial k_t}\left(\tau - \frac{\partial f(a_t^*)}{\partial a_t^*}\right) - \lambda \frac{\partial J(t, k_t - \eta(e_t + (1-\beta) a_t)\tau A k_t)}{\partial(k_t - \eta(e_t + (1-\beta) a_t)\tau A k_t)} \eta(1-\beta)\tau = 0$$

and the FOC with respect to $e_t$ is

$$\frac{\partial J(t, k_t)}{\partial k_t} - \lambda \frac{\partial J(t, k_t - \eta(e_t + (1-\beta) a_t)\tau A k_t)}{\partial(k_t - \eta(e_t + (1-\beta) a_t)\tau A k_t)} \eta = 0.$$

The comparison between the two last FOCs gives

$$a^* = (f')^{-1}(\tau\beta),$$

in which $(f')^{-1}$ is the inverse of the derivative of the function $f$.

For computing the other two variables, instead, we must know the functional form of the value function. The guess function is

$$J = F^\delta \frac{(k_t - H)^{1-\delta}}{1-\delta},$$

in which $F$ and $H$ are constant that will be obtained from the HJB equation. Given this function, the optimal values for evasion and consumption are

$$c_t^* = c_m + \frac{k_t - H}{F},$$

$$e_t^* = \frac{k_t - H}{\tau\eta A k_t}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) - (1-\beta) a_t^*.$$

Once $a_t^*$ and $e_t^*$ are substituting into the HJB we get:

$$
\begin{aligned}
0 =& F^{\delta}\left(k_t - H\right)^{1-\delta}\frac{\delta}{\delta-1}\left(\frac{\rho+\lambda}{\delta} + \frac{\delta-1}{\delta}\frac{1}{\eta} + \frac{\delta-1}{\delta}\left(1-\tau\right)A - \frac{1}{\eta}\left(\lambda\eta\right)^{\frac{1}{\delta}} + \frac{\delta-1}{\delta}\left(\tau\beta a^* - f\left(a^*\right)\right)A - F^{-1}\right) \\
&+ F^{\delta}\left(k_t - H\right)^{-\delta}\left(-c_m + \left(\tau\beta a^* - f\left(a^*\right) + \left(1-\tau\right)\right)AH\right).
\end{aligned}
$$

This function can be split into two equations: one which contains the terms with $\left(k_t - H\right)^{1-\delta}$ and one which contains the terms with $\left(k_t - H\right)^{-\delta}$. Thus, we get

$$
\begin{aligned}
F^{-1} =& \frac{\rho+\lambda}{\delta} + \frac{\delta-1}{\delta}\frac{1}{\eta} + \frac{\delta-1}{\delta}\left(1-\tau\right)A - \frac{1}{\eta}\left(\lambda\eta\right)^{\frac{1}{\delta}} + \frac{\delta-1}{\delta}\left(\tau\beta a^* - f\left(a^*\right)\right)A, \\
H =& \frac{c_m}{A\left(\tau\beta a^* - f\left(a^*\right) + \left(1-\tau\right)\right)}.
\end{aligned}
$$

# B Laffer Curve

When $c_m = 0$ the derivative of government income w.r.t. tax is

$$
\left.\frac{1}{dt}\frac{\partial \mathbb{E}_t\left[dT_t\right]}{\partial\tau}\right|_{c_m=0} = \left(1 - \beta a_t^*\right)Ak_t - \tau\beta\frac{\partial a_t^*}{\partial\tau}Ak_t,
$$

whose second derivative is

$$
\left.\frac{1}{dt}\frac{\partial^2 \mathbb{E}_t\left[dT_t\right]}{\partial\tau^2}\right|_{c_m=0} = \beta\left(-2\frac{\partial a_t^*}{\partial\tau} - \tau\frac{\partial^2 a_t^*}{\partial\tau^2}\right)Ak_t.
$$

If this derivative is always negative, then the curve has a unique maximum. Since $f\left(a\right)$ is increasing and convex, then $\left(f'\right)^{-1}$ is increasing, which means that $a^*$ is increasing in $\tau$. The second derivative of $a^*$ w.r.t. $\tau$ depends on the sign of the third derivative of $f\left(a\right)$, which has not been defined.

In the case of a power function

$$
f\left(a\right) = f\left(0\right) + \omega a^{\gamma},
$$

with positive $\omega$ and $\gamma > 1$, the second derivative is always negative. In this case, in fact

$$
a^* = \left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}},
$$

from which

$$
\begin{aligned}
\frac{\partial a_t^*}{\partial\tau} &= \frac{1}{\gamma-1}\frac{1}{\tau}\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}} \\
\frac{\partial^2 a_t^*}{\partial\tau^2} &= \frac{1}{\gamma-1}\left(\frac{1}{\gamma-1} - 1\right)\frac{1}{\tau^2}\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}
\end{aligned}
$$

and so

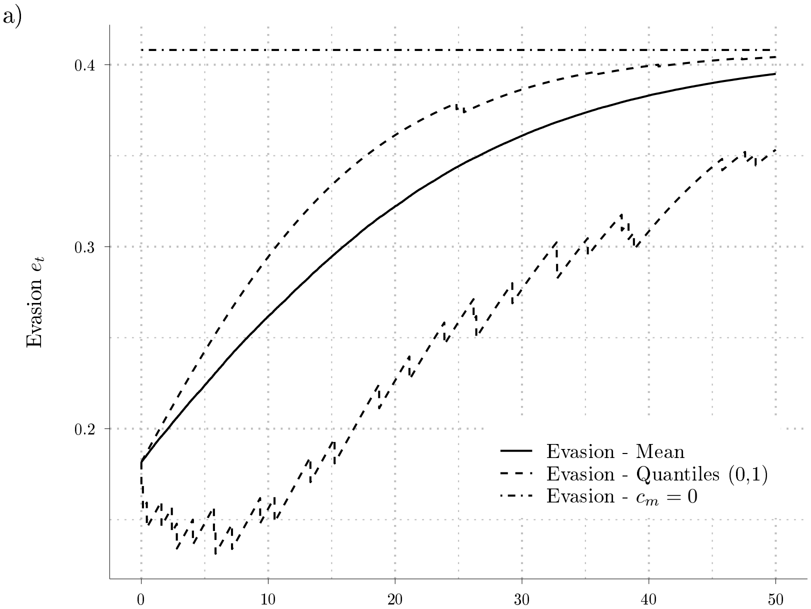
$$\frac{1}{dt}\frac{\partial \mathbb{E}_t\left[dT_t\right]}{\partial \tau}\bigg|_{c_m=0} = \left(1-\frac{\gamma}{\gamma-1}\beta\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}\right)Ak_t$$

$$\frac{1}{dt}\frac{\partial^2 \mathbb{E}_t\left[dT_t\right]}{\partial \tau^2}\bigg|_{c_m=0} = -\beta\frac{1}{\tau}\frac{\gamma}{\left(\gamma-1\right)^2}\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}Ak_t < 0.$$

So, if the function $f\left(a\right)$ is a power function, there is only one maximum.

Figure 1: a) Evasion dynamics b) Ratio of consumption to capital dynamics

a)




b)